Handout 02/04

Topic "Passport to Advanced Mathematics"

1. Find the minimum of the following equation: $y = x^2 - 13x + 40$

2. What are x intercepts of the parabola given by the equation $y = 2x^2 - 8x - 42$? What are the coordinates of its vertex?

3. You buy 120 feet of fence to enclose a rectangular gardening space. What is the area of the largest space you can create?

4. A gym chain currently has 1800 members and charges a monthly membership fee of \$20. Corporate is thinking of raising their fee but estimates that they will lose 40 members for every 50 cents they increase the membership fee. What membership fee should the gym charge to maximize its profits?

5. A rectangular playpen in a dog park has two identical sections divided by a fence. 240 feet of fence are used to form both the outside fence and the dividing line. What dimensions (length and width) should be used to create the largest playpen?

```
6. Simplify the expression: 5^{3/2} \left(\frac{1}{25}\right)^{-3/8}
```

7. If 8q - 6z = 4, what is the value of $\frac{27^z}{9^{2q}}$?

8. What is the average of the all solutions to the equation $-3 = \sqrt{4r + 29} - 2r$?

9. In the system of equations given below, what is the value of 14y - 10x?

$$2x - y + 18w = 4z + 10$$
$$2x - 4y - 16 = -\frac{8}{3}z + 12w$$

10. A sequence of numbers is given by the equation below:

$$R(n) = (-2)^n (n^2 + 10n - 5)$$

What is the sign of the 2023rd term of this sequence?

11. Let
$$f(x) = \frac{1}{x^2}$$
 what is the value of $f(f(2))$?

12. A bacteria colony currently has 250 organisms. Compose an exponential growth model using time t in hours for the following different scenarios. Remember, exponential models on the SAT have the form $a(1+r)^{ct}$, where a is the initial amount, r is the growth rate, and c is the relative time factor:

a) The number of bacteria doubles every hour

b) The number of bacteria increases by 50% every 4 hours

c) The number of bacteria triples every 12 hours

d) The number of bacteria increases by 10% every 20 minutes

13. The bacteria colony from the previous question has reached an asymptotic count of 100,000 organisms and its growth has stalled. Now, it is beginning to exponentially decay. Compose an exponential <u>decay</u> model, still using time t in hours, for the following scenarios:

- a) The bacteria half-life is one hour
- b) The half-life is now 6 hours
- c) The number of bacteria decreases by 35% every 7 hours
- d) The number of bacteria decreases by 0.5% every 6 minutes